California Institute of Integral Studies

International Journal of Transpersonal Studies

Volume 39
Issue 1 *Iss. 1-2 (2020)*

Article 7

9-1-2020

A Fractal Epistemology for Transpersonal Psychology

Terry Marks-Tarlow

A Fractal Epistemology for Transpersonal Psychology

Cover Page Footnote

N/A

A Fractal Epistemology for Transpersonal Psychology

Terry Marks-Tarlow
Private Practice
Santa Monica, CA, USA

The role of science has been controversial within the nascent field of transpersonal psychology. Traditional linear and reductionist models are insufficient to address rare and unreproducible states of mind, fringe rather than normative experiences, and highly personal or culturally specific aspects of awareness. Through a fractal epistemology this paper introduces novel metaphors, models, and methods within a more holistic, organic, and synthetic branch of science. Principles of the epistemology illuminate observer dependence, fuzzy boundaries, recursive patterns, and higher dimensional phenomena that emerge within the infinite expanses between ordinary, finite (Euclidean) dimensions.

Keywords: ***fractal geometry, complexity theory, reductionism, holism, metaphor, boundaries, nonlinear science, implicit awareness, embodiment***

Since the inception of transpersonal psychology in the late 1960s, controversy has surrounded the scope and definition of the field (see Lajoie & Shapiro, 1992). When it first emerged, transpersonal psychology aimed to transcend limitations of research and methods available to conventional psychologists. Spawned within the humanistic movement, the title of Maslow's 1971 book expresses his desire to explore *The Farther Reaches of Human Nature*. An early goal was to reject a narrow focus on psychopathology in order to deal with the whole person. Much of Maslow's early work involved documenting peak experiences, altered states of consciousness, and various spiritual dimensions of life.

From the beginning, these highly subjective phenomena of interest to transpersonal psychologists possessed the characteristics of being ineffable and ambiguous. They often involved rare and unreproducible states of mind, fringe rather than normative experiences, and aspects of awareness that are highly personal and culturally specific. Beyond difficulties in quantifying these sorts of phenomena, an identity confusion has pervaded the subfield of transpersonal psychology, within academic psychology at large. Are these social or "soft sciences" primarily qualitative, descriptive endeavors akin to the humanities, or are they quantitative, empirical under-takings more like chemistry and other "hard sciences"?

Theorists, such as Walsh (1992) suggested an inclusive, integrative strategy, whereby all approaches to transpersonal phenomena are recognized as valuable. Likewise, Wilber (1983) argued for an expanded epistemology that includes sensory, mental-phenomenological, as well as contemplative data. To these folks, any single approach, no matter how objective-seeming, is nonetheless only partial, limited, and unable to capture the whole truth. On the other side of the transpersonal divide, Friedman (2002, 2013) argued for greater scientific rigor. From his point of view, "theories of everything," such as that of Wilber, which arise out of a fully inclusive attitude water everything down so fully as to explain nothing. Friedman proposed that the designation of "transpersonal psychology" be reserved for only that which can be empirically studied, while purely subjective phenomena and fringe disciplines, like astrology, be gathered under the looser designation of "transpersonal studies." As the debate rages on, Jorge Ferrer (2014) counter-argued that Friedman's supposedly "objective" lens of strict empiricism is guilty of its own charge: since no perfectly objective stance exists, every perspective is fraught with its own set of assumptions and biases.

https://doi.org/10.24972/ijts.2020.39.1-2.55

The aim of this paper is to present an epistemology for the field of transpersonal psychology that helps to heal an ever-widening schism between these two positions. To honor the call for objective rigor, I offer up the mathematics of fractal geometry as model, method, and metaphor for otherwise ambiguous and inaccessible transpersonal phenomena. To preserve the breadth and richness of personal and cultural phenomena called for by more inclusionary approaches, I suggest that this nascent mathematical field provides a wider framework than conventional empirical approaches from which to consider even the most unique and subjective of mental states, as well as to tackle the complex interrelationship between subjective and objective realms.

To choose a branch of mathematics as an epistemological framework could be powerful, because there are clear underlying assumptions, plus unambiguous "right" and "wrong" answers for many, if not most, mathematical problems. For multiple reasons, mathematics is often considered the most rigorous discipline of all. What is more, quantitative experiments within any subfield of psychology rely upon mathematics at their foundation, usually in the form of statistics.

Yet, despite this reputation for rigor, Lakoff and Núñez (2000) argued that even math has no objective origins. In their book, *Where Mathematics Comes From*, these researchers argued that mathematics is instead a fully embodied discipline emerging from the movement of our bodies as they interact in a physical world. Lakoff and Núñez pointed out metaphorical origins for even as basic a concept as "number," which can be conceived in multiple ways, depending upon which metaphor is chosen. Whether considered a collection of objects, a member of a set, or a point on a line, this has important implications, with entailments leading not only to wholly different branches of mathematics, but also at times, to contradictory assumptions among these various branches.

Since his discovery/invention of fractal geometry during the 1970s, Benoit Mandelbrot considered this new branch to be the mathematics best suited to understanding features of the natural world. In fact, in *The Fractal Geometry of Nature,* his manifesto published in 1977, Mandelbrot offered fractals as a framework for modeling aspects of nature previously considered too ambiguous, irregular, unique, discontinuous, or complicated for traditional mathematical methods. Over the past 50 years, tens of thousands of researchers have used fractal geometry to model every facet of nature, from microscopic patterning within the quantum realm to the cosmic patterning of galaxy clusters, as well as everything in between, at the mesoscopic level.

By assigning quantitative number (in the form of fractal dimension) to qualitative aspects, fractal geometry is ideal for understanding natural features like the *fluffiness* of clouds, the *jaggedness* of a shoreline, or the *ruggedness* of a mountain range. This mathematical power to model complicated patterns extends from outside to inside the human body. Pioneer nonlinear researchers such as West (2013) and Liebovitz (1998) documented how fractal patterns pervade the complicated physiology of our lungs, circulatory system, and neural structures. Other examples of its utility include fractal measurement to differentiate tumor from normal cells (Baish & Jain, 2000), as well as the visual productions of famous artists who suffered from degenerative brain conditions versus those who did not (Williams & Reilly, 2016).

In my own work as a clinical psychologist, I have written extensively about the fractal geometry of human nature (e.g., Marks-Tarlow, 1999, 2004, 2008, 2010, 2011, 2012, 2015). I believe that nonlinear science broadly, and fractal geometry specifically, provide a holistic, flexible meta-framework for understanding even the most complex psychological, social, cultural, and historical systems. Because fractal patterns extend across space, time, as well as symbolic realms (Schroeder, 1991), fractals can illuminate complex interrelationships, such as the interpenetration between brain and mind, self and other, or inner versus outer realms.

In sections to follow, I begin with a brief description of the history of fractal geometry, including its uncanny parallels with the early history of transpersonal psychology. I then describe specific features and properties of fractal geometry that are useful for conceptualizing otherwise inaccessible qualities of transpersonal phenomena. This paper ends with a list of principles derived from fractal geometry in hopes of providing a novel epistemology for transpersonal psychology.

What is a Fractal?

Everywhere we look, fractals surround us—in the branching patterns of a tree, the spots of a leopard, or the wrinkles of an elderly human face. Although each of us understands fractal patterns intuitively, in an embodied way, very few of us tend to "see" them consciously. Why is this? An important reason may be because the field of fractal geometry is too new. Few of us have grown up with fractal objects as part our visual or mathematical lexicon. Instead, traditional Western education has privileged linear lenses by highlighting straight lines and regular forms, such as Platonic solids and Euclidean dimensions. It is easy to remember elementary school activities of playing with such shapes—for example, cutting out and pasting a larger triangle onto a smaller rectangular base and calling it a pine tree. Yet, all the while, we could sense our productions as mere approximations of the real thing.

What constitutes the "real" thing? In other words, how do natural shapes differ from human-made ones? Is there an archetypal meta-pattern—that is, a pattern of patterns—that Nature draws upon again and again? The answer appears to be "yes." Nature loves recursively enfolded shapes, i.e., patterns that are repeated again and again on multiple size and/or time scales. When in elementary school, we could have just as easily played with fractals. Had we cut out multiple triangles, each the same shape, but slightly different in size, placing the smallest one atop of a layered series, all laid upon the smaller rectangular base, we would have played with self-similar, fractal objects while producing a more realistic pine tree.

It is ironic that so few of us have developed a conscious awareness of fractals despite our implicit awareness of them, given what may be a fractal stage of most children's art (Marks-Tarlow, 2008), much like Gardner's (1982) tadpole figure (a circle on top of a stick) to represent the human figure. Figure 1 represents an example of fractal art, spontaneously created by my 5 year old daughter. Whether the shape consists of a heart, diamond, or oval, I believe there exists a universal desire in children to play with the same shape on different size scales. Meanwhile, just about every parent recognizes some variation of this drawing within their own children's early art productions.

An important reason fractal play (see Marks-Tarlow, 2010) may be a stage of children's art involves the dynamics of the visual field. As people approach or retreat from babies, similar shapes on different size scales appear and re-appear successively upon the flat surface of our retinas. Objects or people appear larger as they move towards us (or we move towards them) and smaller as they (or we) move away. In this respect, our eyes intuitively understand the multi-scaled quality of fractal dynamics, which works as an algorithm to make sense of our own position relative to people and things in our visual landscapes.

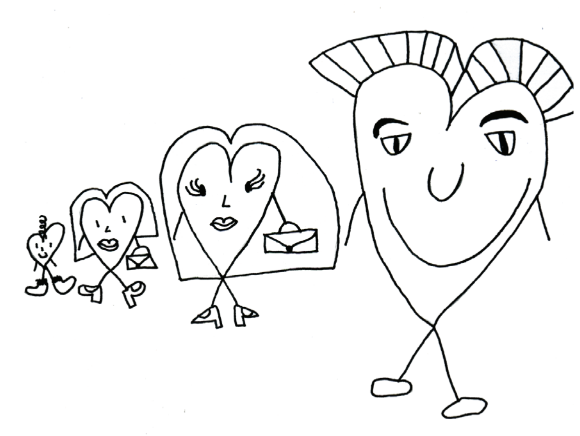

Figure 1. Fractal stage of children's art (Courtesy of Darby Tarlow)

This hallmark property of a fractal, as stated more formally, is called "self-similarity." Within fractal geometry, self-similarity means that the large-scale pattern of the whole gets repeated on multiple size or time scales within its parts. Self-similarity involves recursive, that is self-reflexive, symmetry. A related fractal property is called scale-invariance, which means that the same pattern repeats itself either identically or approximately across multiple size or time scales. Many growth processes are self-similar as well as scale-invariant. Consider the successive growth of a nautilus shell, as illustrated in Figure 2. The mathematical qualities of the algorithmic spiral reveals how the shell's basic shape, or identity, gets maintained by preserving part/whole relations, despite successive changes in size. The numbers inside the boxes—1, 1, 2, 3, 5, 8—are named the Fibonacci sequence, first defined by Euclid and written about in the 15th century by Luca Pacioli, an Italian monk reputedly "drunk on beauty" (Olson, 2006). To get the next number in the series, simply add together the previous two numbers. By dividing each pair of successive numbers, one arrives ever closer to "the golden ratio" (1.61803…). For millennia, the golden ratio number has been capitalized upon in art and architecture, romanticized in literature, and spiritualized under the name of "sacred geometry" (Lawlor, 1982). Because of the self-similar preservation of part/whole relations, the Fibonacci series represents an early recognition of fractals that describes many common aspects of nature—from the reproductive rate of rabbits, to the spirals of a sunflower or helical form of a pine cone.

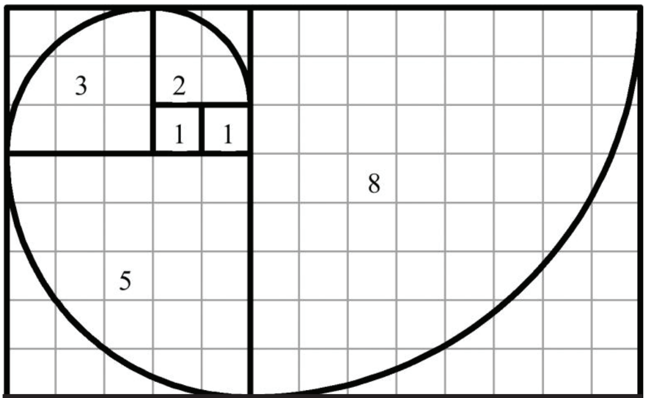

Figure 2. Self-similar construction of a nautilus shell

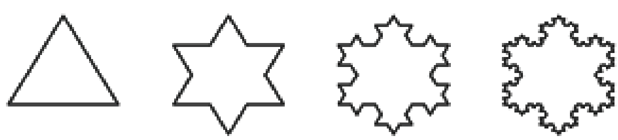

Figure 3. First four iterations of a Koch snowflake

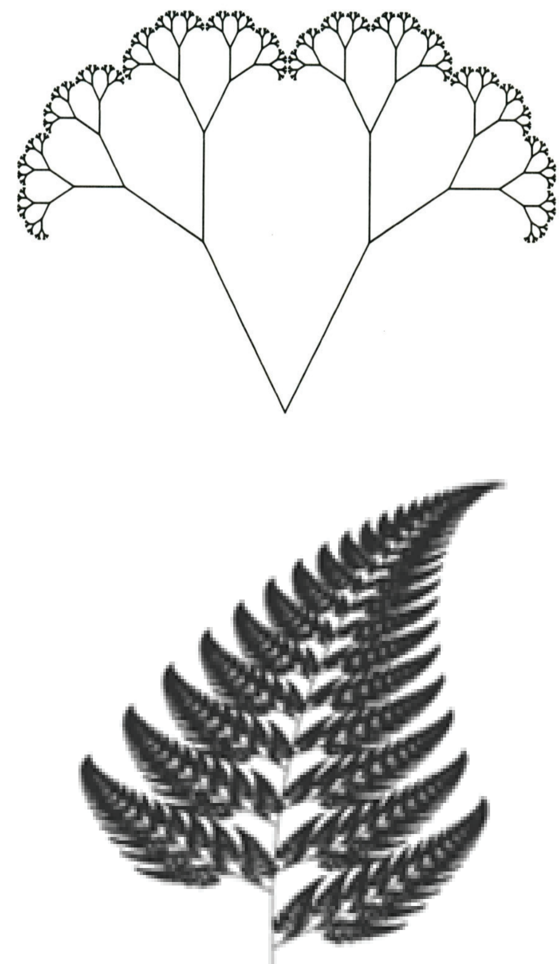

Figure 4. Linear vs nonlinear fractal branching algorithms

Beyond the Fibonacci series, there are many different ways to construct a fractal. One involves applying the same algorithm, or procedure, over and over to a seed shape. Consider the Koch snowflake (see Mandelbrot, 1977) in Figure 3. The seed shape consists of a triangle; the algorithm involves removing the middle third of each side and replacing it with two thirds of a smaller triangle. The figure below reveals the first four stages, or iterations, of this process, which can extend indefinitely, at least in theory, even though at a certain point, our eyes fail to see the tiniest iterations.

Fractals like the Koch snowflake are linear, because the identical pattern is repeated on each size scale. Fractals can also be nonlinear, by tossing a bit of chance or randomness into each iteration. Herein lies a critical difference between the regularity of human-made objects and the irregularity of natural ones. Consider for example the genetic code: despite a single underlying growth algorithm, intensely variable conditions within the environment tweak the resulting epigenetic manifestations, from ever so slightly to quite dramatically. Figure 4 helps to visualize the difference between linear fractals and nonlinear ones.

History of Fractal Geometry

Within the history of mathematics, linear objects and additive methods have prevailed. Consider the invention of calculus in the late 17th century simultaneously by Newton and Leibnitz, designed to capture continuously evolving dynamics of motion (see Figure 5). By chopping the space under a curve into smaller and smaller units, each subsection could be added together to reveal the total area. Using the device of calculus, very complex curves could be broken down and measured. The concept of infinity creeps into the methodology as follows: it lurks implicitly as an idealized point at the limit of the measurement, as the size of units become infinitesimally small by shrinking towards zero.

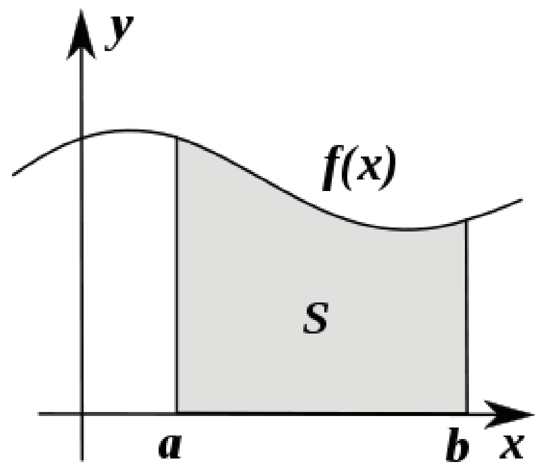

Figure 5. Calculus

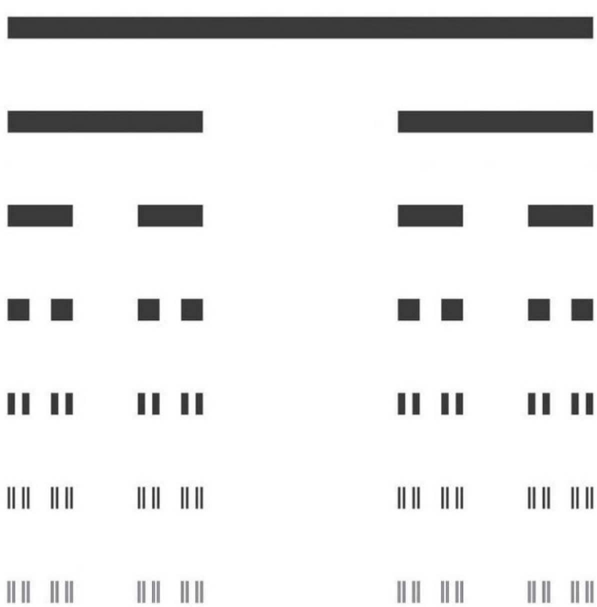

Figure 6. Cantor dust

The concept of infinity plays a more explicit role in the invention of fractal geometry. Consider the mathematician Georg Cantor in the late 19th century, whose work was an important precursor to fractal geometry (Mandelbrot, 1977). Up until Cantor's time, it was assumed that infinity was absolute, that is, that infinity comes in one and only one size. This idealized and definitive view was why some mathematicians equated mathematical productions with the hand of God, whose infinite power was seen as equally absolute. By innovating a method of one-to-one correspondences, Cantor discovered that there are endless "flavors" or sizes of infinity, such as the difference between the set of rational numbers versus the set of irrational numbers. Even bigger is the size of the set of all sets of numbers. By introducing an "infinity of infinities", Cantor gave birth to a new field of "transfinite" mathematics.

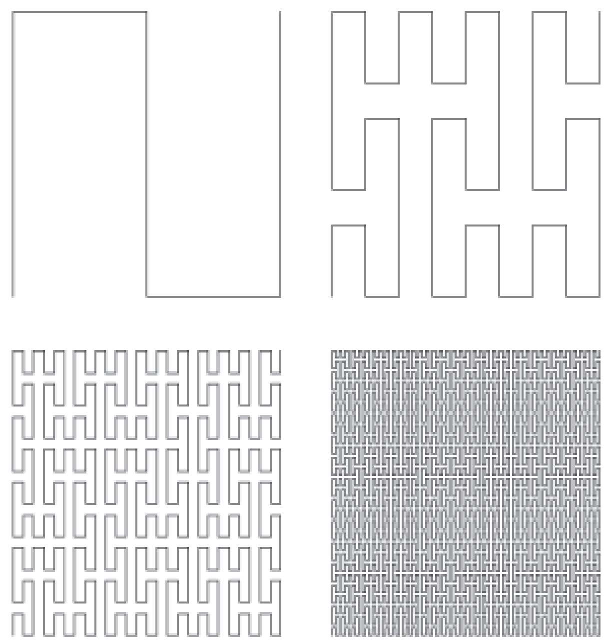

Figure 7. Four iterations of the Peano curve

At the inception of transfinite math, many mathematicians were shocked. Poincaré referred to Cantor's ideas as a "grave disease" that was "infecting" the field of mathematics (Dauben, 1979). Whereas Cantor believed his discoveries had been handed to him by God, some Christian theologians feared Cantor's work challenged the uniqueness of God's absolute infinity. One way to understand Cantor's brand of infinity is to contemplate his fractal contribution, called Cantor dust (see Figure 6). Whereas the Koch snowflake (Figure 3) uses each successive iteration to add structure, Cantor dust uses each successive iteration to remove structure, in this case, the middle third of each line.

Another 19th century iconoclast and precursor to fractal geometry was Guiseppe Peano (Mandelbrot, 1977), whose space filling curve, much like both the Koch snowflake and Cantor dust, is notable because it eludes conventional methods of calculus (see Figure 7). Because the Peano curve possesses no tangents, it is considered "undifferentiable" or outside the scope of calculus.

Mathematical forms produced by Koch, Cantor, and Peano represented new and unconventional objects that were met with high suspicion. At times, mathematical colleagues dismissed these objects as irrelevant. At other times, they rejected them as outliers or deemed them "pathological" and "a gallery of monsters" (Mandelbrot, 1977). Looking back at the revolution of ideas that separated the classical mathematics of the 19th century from the modern mathematics of the 20th century, it is ironic that the very shapes dismissed as irrelevant and rejected as monstrous have proven over time to conform most highly to Nature's recursive patterning.

Within the history of transpersonal psychology, Stanislav Grof (2008) documented the predilection of mainstream psychologists and psychiatrists to similarly reject and psychopathologize transpersonal phenomena of interest. As a trend, Western materialistic scientists easily dismiss the realm of spirituality as reflections of mere ignorance, gullibility, superstition, self-deception or primitive magical thinking. Meanwhile, the experiences of visionaries, prophets, or saints at the root of the world's major religions are frequently seen to be indicative of serious mental illnesses. In the words of Grof:

> St. Anthony has been called schizophrenic, St. John of the Cross labeled a "hereditary degenerate," St. Teresa of Avila has been dismissed as a severe hysterical psychotic, and Mohammed's mystical experiences have been attributed to epilepsy.… Franz Alexander (1931), known as one of the founders of psychosomatic medicine, wrote a paper in which even Buddhist meditation is described in psychopathological terms and referred to as "artificial catatonia" (pp. 47–48).

Perhaps these parallels between the early days of fractal geometry and those of transpersonal psychology are less coincidental than they might seem. While mainstream mathematics was busy addressing conventional issues, Cantor, Peano and Koch were examining fringe ideas. In parallel fashion, while mainstream psychology was following its own set of normative trends, transpersonal psychologists were also drawn towards the fringes. Grof asserted ontological realism for transpersonal experiences of the interconnection between all beings and levels of existence, an idea dismissed primarily by reductionist scientists. Perhaps pioneers in fractal geometry and transpersonal psychology were rejected as unconventional, even heretical, largely from the perspective of reductionist science. Perhaps the two fields share a similar history because through more holistic, integrative lenses, they both model the same thing—what is unique, irregular, and rare in nature, including human subjective experience.

There is a famous paper by the mathematician Wigner (1960) entitled, "The unreasonable effectiveness of mathematics in the natural sciences." Wigner's focus was primarily on amazing correspondences between mathematical formulae and outer physical realms of the material level. Perhaps we are on the cusp of a transformation by perceiving the unreasonable effectiveness of mathematics within the social sciences. In the spirit of Wigner, I suggest transpersonal psychology is in need of a more holistic scientific/mathematical fractal framework that helps to embrace the full breadth and depth of its psychological and experiential scope.

Fractional Dimensionality: The Endless Space *between* Dimensions

In order to understand how fractals model identity in nature, as well as provide a bridge between various realms of space, time, and imagination (Marks-Tarlow, 2004, 2012), it is important to examine how fractals illuminate certain aspects of subjective experience. Where does consciousness begin? Where does it end? What are its bounds, especially given that the subjective *feel* of conscious awareness seems to extend *across* boundaries (from inside our heads to outside our bodies)? How does the invisible substance of consciousness relate to the materiality of our brains and bodies? What is the difference between an objectively measurable event and a subjectively held experience?

All remain disputable issues often relegated to the realm of philosophy. A complete answer to these questions is beyond the scope of this paper. However, before highlighting a couple of issues relevant to this discussion, I begin with a disclaimer. In sections that follow, including the list of epistemological principles at the end of this paper, I do not claim to have solved what is labeled the "hard problem" of consciousness, as formulated by David Chalmers. In his words (Chalmers, 1995, p. 201):

> It is undeniable that some organisms are subjects of experience. But the question of how it is that these systems are subjects of experience is perplexing. Why is it that when our cognitive systems engage in visual and auditory information-processing, we have visual or auditory experience: the quality of deep blue, the sensation of middle C? How can we explain why there is something it is like to entertain a mental image, or to experience an emotion? It is widely agreed that experience arises from a physical basis, but we have no good explanation of why and how it so arises. Why should physical processing give rise to a rich inner life at all? It seems objectively unreasonable that it should, and yet it does.

Chalmers formulated the hard problem as difficulty explaining the *contents*, or *qualia*, of conscious experience. He outlined and then dismissed the success of various case studies proclaiming to explain consciousness, including Crick and Koch's (1990) suggestion that gamma oscillations in the cerebral cortex provide the neurobiological correlates of consciousness or Penrose's (1994) suggestion from within a nonlinear dynamics perspective that nonalgorithmic processing explains mathematical reasoning. I want to clearly state that I am not presenting a theory of conscious awareness or an explanation of how we come to experience its various qualia. Instead I offer fractal geometry as a means of modeling some features that pertain to the *structure* of subjective experience, including the possibility of open boundaries between conscious awareness and physical, material levels of brain, body, and surrounding environment.

Keeping these limitations in mind, I turn next to an important distinction between objectively measurable events and subjective experience. Objectively measurable events are discrete and observable. In order to measure something, it must have clear boundaries plus a clear value within those finite bounds. Subjective experience, by contrast, carries the feeling of being immeasurable and infinitely deep, with borders that feel fuzzy and ambiguous. Perhaps there is a unified field of consciousness—a truly transpersonal extension of invisible subjective dimensions into objective realms. Shamans who claim to transport themselves through their astral bodies would certainly be such a case. The true potential of consciousness remains unknown. But again, most relevant to this discussion is the subjective feeling of fuzzy boundaries and infinite extension, both during contemplation of inner worlds as well as perception of external worlds. This very sense of boundary-less interconnection and complete interpenetration of inside and outside realms corresponds to mystical experiences and peak states like "nondual" awareness, whether facilitated by psychedelic substances or occurring naturalistically.

How does a mystical sense of infinite extension relate to fractals? When I first came across this new branch of geometry in the early 1980s, I immediately had the intuition that there is something profound about fractals. At the time, I was attending a weekly drawing group that included the physicist Richard (Dick) Feynman, and we had become quite close. Because Feynman was deemed the smartest man in the world (after Einstein), I rushed to him with the question, "Don't you think fractals are profound?" Someone standing nearby asked what a fractal is. Dick took several minutes to give a state of the art explanation of fractal geometry—its hallmark features of self-similarity, scale invariance, and more. I waited patiently, and once Dick had finished, I asked him again, "Don't you think fractals are profound?" His response—"I don't understand them"—absolutely shocked me. How could this be, when Feynman had just explained fractals so eloquently?

Crestfallen, I was left utterly alone to find my way forward. It has taken me decades to flesh out my understanding of fractals, including 15 years to write *Psyche's Veil* (Marks-Tarlow, 2008), which applies chaos theory, complexity theory, and fractal geometry to clinical practice. To this day, I still don't understand fractals fully, nor do I believe I ever will. But one thing I am quite certain of—my initial feeling of profundity relates to the role infinity plays within fractal construction. Let me explain.

Ordinary Euclidean dimensions are finite, that is, they consist of whole numbers such as integers. Points are 0 dimensional (0-D); lines are 1 dimensional (1-D); planes are 2 dimensional (2-D); solids are 3 dimensional (3-D). Einstein offered time as the 4th dimension, while others view imagination as the 4th dimension (see Marks-Tarlow, 2008). Human made objects, such as the top of a table, have clear boundaries within the confines of finite Euclidean dimensions. The measurement of a table's circumference is resolvable—we always arrive at the same approximate answer, no matter how large or small our measuring device. Whether our ruler is 6 inches long or 6 feet long, the measurement of a table's circumference remains essentially the same.

None of these conditions apply to fractals, which are multi-scaled objects that are not finite, but infinitely deep, at least in theory. Because of the properties of being multi-scaled and infinitely deep, fractals do not have clear boundaries. Their measurement is not fixed, but is fuzzy and dynamical instead. To illustrate this, consider the Mandelbrot set (Figure 8), the granddaddy of all fractals and the most complex mathematical object known to humankind (Dewdney, 1985). In order to construct the Mandelbrot set, the same formula, $f(z) \rightarrow z^2 + c$, is iterated for every point on the complex number plane. Iteration means that the end product of an equation is fed back into the beginning over and over again, that is, recursively, until the equation resolves itself (or doesn't).

In the figure below, the solid black areas represent the finite zone where the formula resolves to a fixed number. The white areas represent the infinite zone where the formula goes on and on, as it extends towards infinity. The complex border between these two zones represents the dance of the Mandelbrot's intricate, multi-scaled pattern. This edge of complexity is infinitely deep. This means that when the computer is used as a "microscope" to zoom in on a particular area, ever new pattern emerges dynamically and unpredictably. Figure 8 reveals 4 scales of zoom on the Mandelbrot set's complex edges. Notice the self-similarity that re-appears in the fourth square, such that the very similar shape of the whole reappears, making it quite difficult to tell what is inside and what is outside its borders.

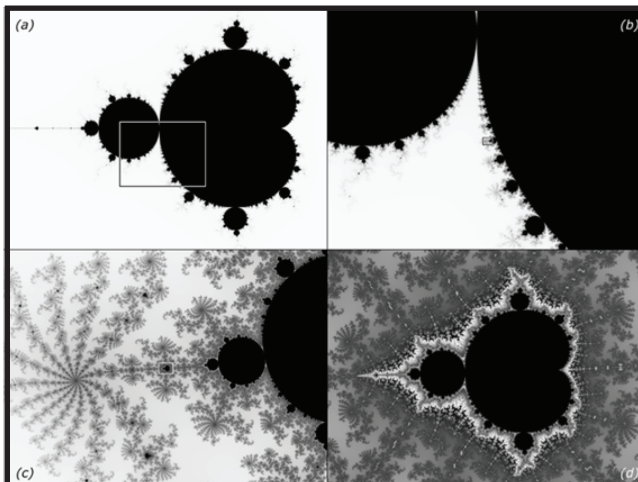

Figure 8. The Mandelbrot set, $f(z) \rightarrow z^2 + c$ (Courtesy of Nicolas Desprez)

From this example, we can see that fractal geometry is a very visual form of mathematics that is intimately dependent upon the prodigious calculating power of the computer. This fact helps explain why fractal geometry was not discovered until the 1970s. Fractal zooms abound on YouTube,

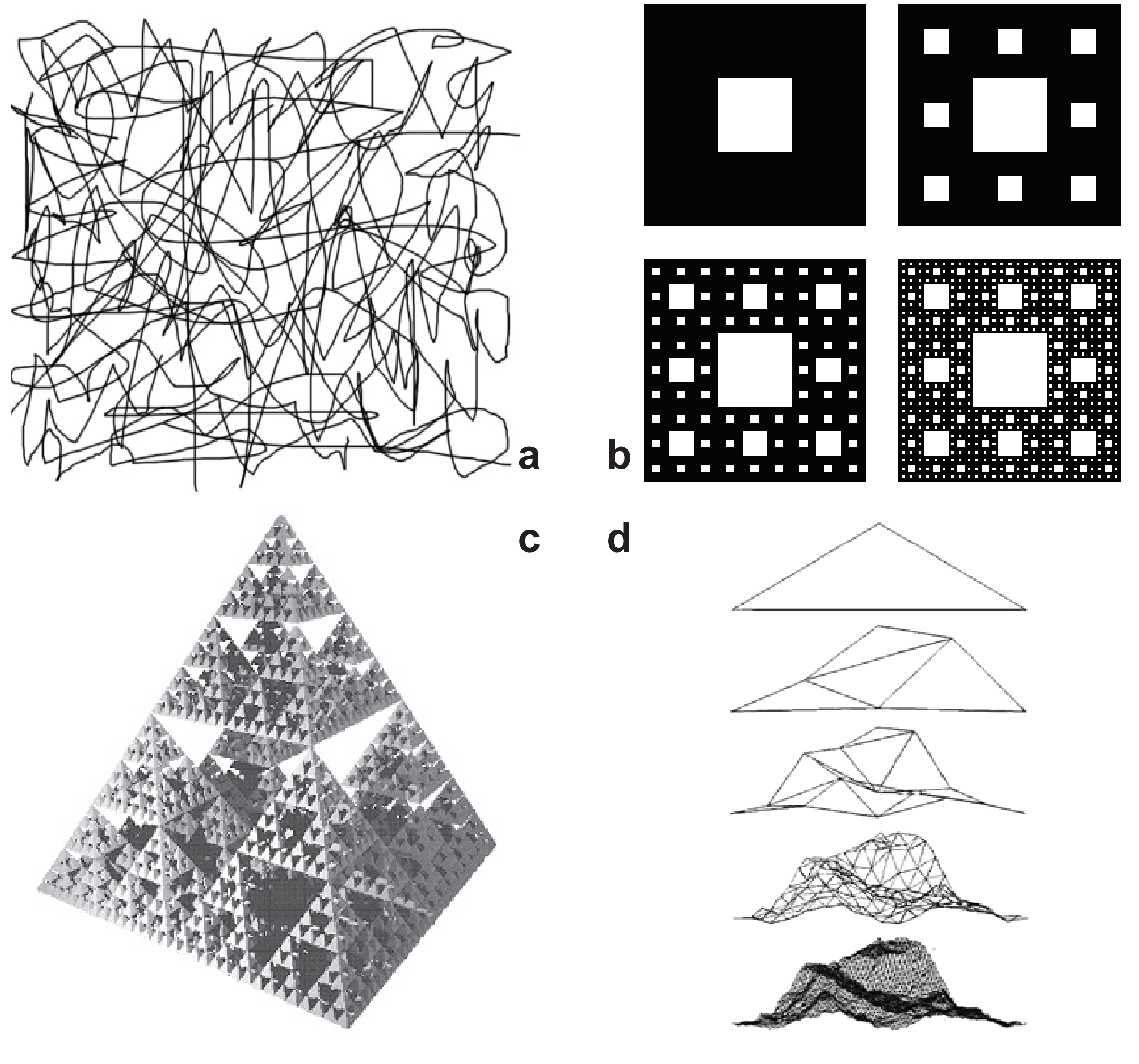

Figure 9. Fractal dimensionality – a) Scribble, b) Sierpinski carpet, c) Sierpinski triangle, d) Fractal mountain

and it is highly illustrative to watch a few of these short videos to get a feel of for the endless beauty and depth of fractal geometry. To more fully understand the infinite aspect of fractal geometry, I next examine the concept of fractal dimensionality. Contrary to ordinary assumptions, fractals grow in the endless space *between* finite Euclidean dimensions. Mathematically, the discovery of fractals required expansion of the very notion of dimensionality, such that each mathematical fractal not only has a discrete Euclidean dimension, but also has a fractal dimension, consisting of a fractional number that carries the potential of being infinitely variable.

Figure 9 illustrates how ordinary scribbles as well as more formal Sierpinski carpets occupy the space between a 1-D straight line and a 2-D plane (Figure 9a, 9b). Meanwhile Sierpinski pyramids and ordinary mountains occupy the territory between a 2-D plane and a 3-D space (Figure 9c, 9d).

In general, no matter what the Euclidean dimension, the higher the fractional dimension,

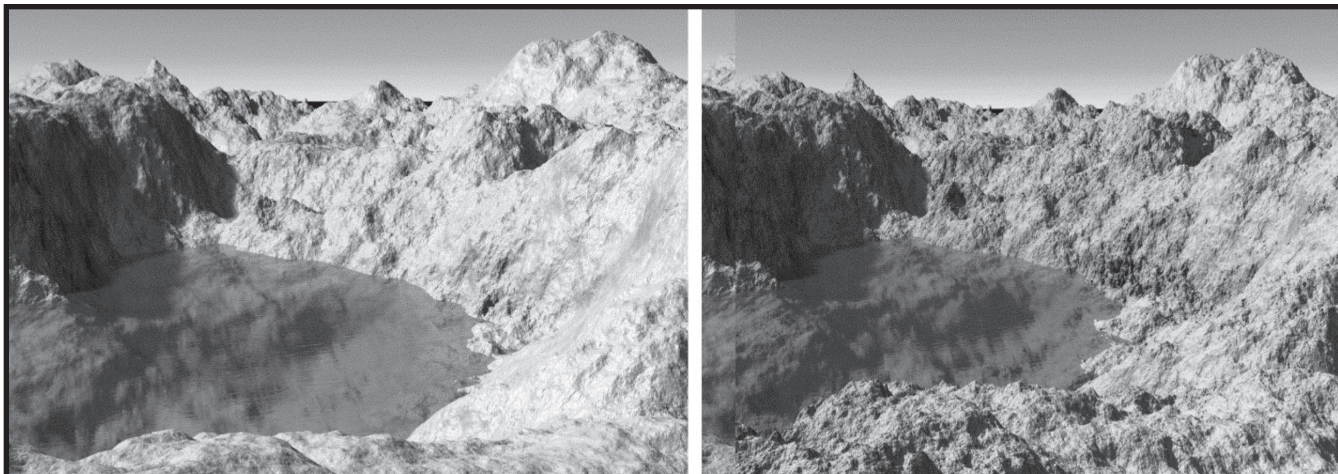

Figure 10. The same fractal mountain scape—with lower fractal dimensionality (left), and higher fractal dimensionality (right). (Courtesy of Nicolas Deprez)

the more complex the fractal object. Figure 10 shows the same fractal mountain scape rendered in lower versus higher fractal dimensionality. We can now begin to see how fractals help us to quantify qualitative features of Nature, like the ruggedness of a mountain scape, the jaggedness of a coastline, or the fluffiness of a cloud. In an interesting recent application, the fractal dimension of Rorschach test figures was quantified (Abbott, 2017). Despite initial speculations that Rorschach dimensional complexity would mimic that of Nature (e.g., cloud patterns that resemble Mickey Mouse or a submarine), the Rorschach figures are relatively lower dimensional than Mother Nature, revealing a slightly different "fractal sweet spot" that is best suited to the projection of visual imagery from imagination.

Fractal Paradoxes

Mandelbrot (1967) posed a now famous question, "How long is the coastline of England?" At first blush, the answer might seem straight forward. Yet because of the multi-scaled quality of a coastline's fractal shape, paradox lurks within, connected to the construct of fractal dimensionality. Mandelbrot claimed that the length of the coastline of England is infinitely long, and what is more, every other natural coastline is also infinitely long, along with any arbitrarily short subsection of coastline! Mandelbrot's assertion emerges from fractals as multi-scaled objects. The property of infinite depth renders a single, definitive measurement impossible. Instead, the number we arrive at depends intimately on the size of our measuring device. Counterintuitively, the smaller the ruler, the *larger* the number. This resulting quality of observer dependent measurement (Marks-Tarlow, 2008) is illustrated in Figure 11.

Notice that when the ruler is 6 inches long, it is too crude to capture any detail of the Koch curve. When the ruler is 2 inches long, it is short enough to capture more detail, and the length measured extends to 8 inches. As the ruler shrinks to half an inch, the measurement captures yet more detail and extends to 12 inches. Two important additional observations: 1) Even at half an inch long, the measuring stick still doesn't capture all the detail of the Koch curve; 2) Generally, the shorter the measuring stick, the longer the measurement, such that at the mathematical limit of an infinitely small measuring stick, we obtain an infinitely long measurement. Hence, Mandelbrot's claim regarding the infinite length of any section of coastline.

In both the Koch curve example and Mandelbrot set, we see how infinity quite literally exists at the edges of fractal objects. This helps us to grasp how fractals can model irresolvable seeming subjective boundaries. To get a fuller feel for fractal boundaries, such as I claim represent the psychological edges of the self, consider figure 12, which illustrates Newton's method of approximation. Each colored area converges towards one of four correct solutions to the simple equation, $X^4 - 1 = 0$. Each solution consists of a black circle within the center of each of four quadrants, more technically known as basins of attraction. Whereas each solution is finite, the boundary zone separating the four basins of attraction is infinitely deep. What is more, this mathematical rendering also reveals the infinite interpenetration between the parts and the whole. This is because each of the fractal boundary zones contains all of the other basins recursively, an infinite number of times.

The notion of interpenetrating boundaries, such as exists interpersonally, that is, between one person and another, is a subject of great interest to me as a clinical psychologist. I have written about the relational unconscious (Marks-Tarlow, 2008), as shared between therapist and patient, beneath the level of conscious awareness. For example, in *Psyche's Veil* I cite the case of a patient who one day brought into our session *my own childhood dream*. This tidal wave dream was very different from anything she had ever remembered dreaming, as most of my patient's dreams involved scary chases and attacks. Especially in light of the flood of change that happened next, both of us experienced this dream as an unconscious bid to break the enactment stalemating our psychotherapy for several months.

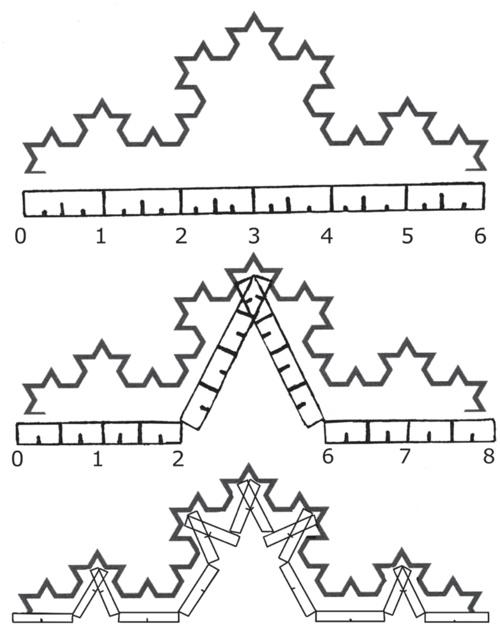

Figure 11. Fractal relativity of measurement. (Courtesy of Terry Marks-Tarlow)

The notions of fuzzy, interpenetrating boundaries between self and other, mind and brain, and brain and body, is consistent with the work of Scott Kelso (e.g., 1997, Kelso & Engstrøm, 2006). Kelso is a nonlinear researcher also interested in how science and philosophy intermingle. He has studied and written extensively about coordination dynamics, that is, how patterns of coordination form, dissolve, adapt, and change through processes of self-organization. When examining implications of coordination dynamics for the brain~mind and brain~behavior barriers, Kelso uses the tilde to symbolize the dynamic nature of complementary pairs, whose polar ends are not only of significance, but everything in between. Kelso has also studied how dynamical patterns of muscular motion existing within one person extend to others, such as when people fall into lockstep or when musicians coordinate so precisely as to anticipate each other's next moves. Kelso's recent work on hyperscanning (Kelso, Dumas, & Tognoli, 2013) extends these examinations even further. Hyperscanning involves the simultaneous brain scanning of two individuals' as they interact in real time. This fascinating line of contemporary neuro-research reveals very little difference between intrapersonal and interpersonal communication. In other words, how messages are sent from one part of the brain to another share similar coordination dynamics to how messages are sent between brains. Such research points towards fluid, dynamic boundaries between self and other, inner and outer realms.

Figure 12 provides a visual representation of fluid boundaries between inner and outer realms in the case of mathematical intuition—subjective guesses at objective answers. Here is how Newton's method of approximation works. In order to address the equation, $X^4 - 1 = 0$, begin with a random guess at a solution, then calculate the formula using your guess as the starting point. How close your guess is to one of four actual solutions determines what happens next. The closer your beginning guess is to an actual solution, the quicker you arrive at the solution; if your initial guess diverges too far, it will land within the chaotic boundary zone *between* solutions, from which there is no exit.

This visualization is particularly interesting in light of paranormal intuition as an important subject of interest within the field of transpersonal psychology (Daniels, 1998). I have also used the diagram to model the chaotic boundaries that so often surround people diagnosed with Borderline Personality Disorder (BPD). As a clinician who works frequently with people suffering from this diagnosis, I can attest to the frequent feeling of loose boundaries that trigger my falling into dangerous, double bind territory, from which there is no escape. I am damned if I do, and damned if I don't, from the perspective of the other—and utterly helpless to assert my own independent perspective. Finally, I would like to suggest another excellent use of figure 12 for re-conceiving Ken Wilber's integral grid. To add fractal boundaries in place of straight lines between his four quadrants increases the power of his model for understanding interpenetrating subjective, intersubjective, objective, and interobjective realms.

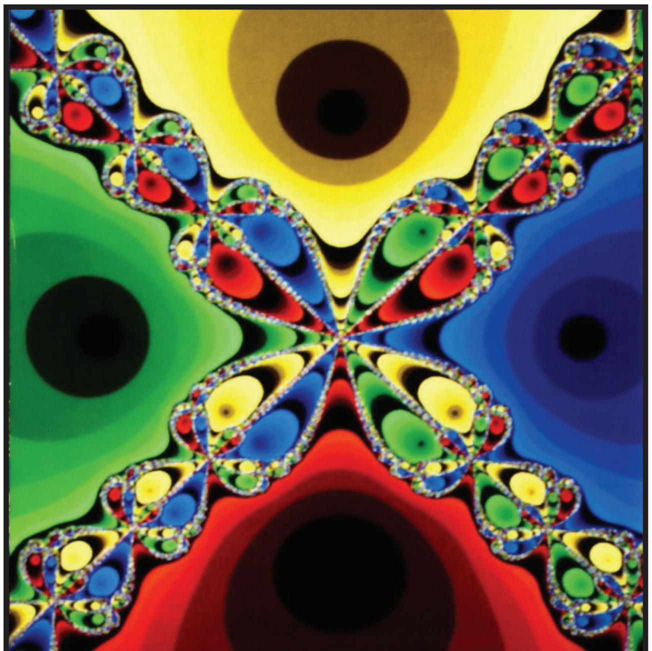

Figure 12. Newton's method of approximation

Power Laws: A New Kind of Statistic

As mentioned earlier, any applied science depends upon mathematics to supply the necessary rigor for its foundations. "Hard" sciences, such as physics, frequently rest upon predictions supplied by pure mathematical formulas. For example, Einstein needed to delve into the strange, nonlinear world of non-Euclidean mathematics in order to prove his theories of relativity. "Softer" sciences, such as biology or economics, often use statistical methods to test between competing hypotheses.

Across all subfields of psychology and other social sciences, normative statistics are traditionally used. This type of statistics, sometimes called a Bell curve, seek the central tendency, that is, the mean or norm of a population or sample. Unfortunately, normative statistics contain underlying assumptions that often prove to be false within most complex systems, as is described in detail in West's (2016) book on the topic. One underlying assumption that frequently proves false is the requirement that all underlying variables operate independently (or orthogonally) from one another. Having had the honor of writing the foreword to West's book, I relayed a mathematical tale from my youth. At the time, a common statistic was floating around that the average American family had 2 ½ children. What the heck does that mean, I pondered, given that *no* family has 2 ½ children? Because transpersonal psychologists are so often interested in idiosyncratic states and non-repeatable circumstances that have nothing to do with central tendencies, the poor fit between normative statistics and phenomena of interest may be particularly exaggerated within this subfield of psychology.

Fortunately, another mathematic distribution exists, called a power law, which excels for modeling rare, unpredictable and unique events. Power laws are temporal fractals, where statistical self-similarity manifests as scale invariant patterns across multiple time scales. As an example, Mandelbrot and Hudson (2010) applied temporal fractals to model stock market fluctuations. Whether examined over the period of a day, year, or decade, the ups and downs of the market reveal statistically self-similar patterns. With chance and randomness part of natural fractal fluctuations, we begin to understand how fractals help us to model transpersonal phenomena that are fundamentally unpredictable, yet simultaneously ordered.

A good example of a power law distribution in Nature is the frequency of earthquakes of various magnitudes, as measured on the Richter scale, a logarithmic metric. It turns out that the chances of a very large earthquake are very small (sometimes called a Black Swan event); the chances of a tiny earthquake are quite large; and the chances of a medium level quake are medium sized. Much like patterns on the stock market, we cannot predict the specifics at any given point, yet we can determine the coarse grain picture.

With normative statistics organized around a mean score, their power lies in the center, such that all variability tends to get collapsed into a single number at the peak of the Bell curve. With larger and larger sample sizes, normative statistics gain both in certainty as well as in predictive power. By contrast, the power of a power law distribution is not in the center, but in the tails, where rare events exist. This type of statistic allows for unpredictability while preserving variability. What this means is that the larger the sample size, the *greater* the variability one finds. Simply put, the more people you sample, the greater the differences you will find between them. Psychologically, this trend certainly corresponds with my professional experience as a clinician. Although depression is ubiquitous as a symptom, to me no two cases look alike, and if they did, I am probably in the wrong profession.

The ability of power law distributions to predict the occurrence of highly rare occurrences, but not their precise timing, seems invaluable for validating, if not tracking, transpersonal phenomena. Here is an empirical example, related to my 1999 paper, "The Self as a Dynamical System." In this paper, I predicted that changes relevant to the self would follow a power law distribution. Much like earthquakes, this would mean that people rarely experience huge changes relevant to self-concept, but would often experience tiny shifts. Delignières and his French colleagues (2004) decided to test this hypothesis. Twice daily, for 512 days, a small group of subjects rated six subjective dimensions: global self-esteem, physical self-worth, physical condition, sport competence, attractive body, and physical strength. Results indicated that changes in self-esteem, as well as changes in perception of physical self, did indeed reveal a fractal distribution. Each subject demonstrated an array of self-similar fluctuations that possessed a unique fractal dimension exponent. Results confirmed my conceptualization of the self as a hierarchically nested, self-organizing, dynamical system. Subjective research such as this fulfills Friedman's (2013) call for rigor, as well as mirroring his own (1983) research on self-expansiveness as a transpersonal construct.

The Computer as Aid to the Human Eye

In previous sections, I have demonstrated how fractals offer a way to visualize otherwise invisible dimensions, as well as how fractals can model interpenetrating boundaries within highly complex, open systems. We saw how invaluable the computer is in the process, since the entire branch of fractal mathematics depended upon its invention for a complete visualization. Generally, researchers interested in nonlinear dynamics, including the complexity sciences, often utilize computer simulations to model highly complicated systems that contain unpredictably emergent or highly idiosyncratic elements. Computer-aided methods, such as agent based modeling, allow researchers to simulate complex systems, by tweaking underlying parameters (values) and then running the system again to see what happens.

In their book *The Philosophical Computer*, Grim and his colleagues (1998) described their use of the computer to model paradoxical philosophical issues too complex to otherwise visualize. Consider, for example, the self-referential assertion, "This statement is false," known since antiquity as the Liar's paradox. The statement is paradoxical because it is true only if it is false, and false only if it is true. Translated into a mathematical equation iterated by computer, figure 13 shows a way to visualize the paradox as a periodic attractor bouncing back and forth between 2 values: 1 (true) and 0 (false). Logic is ordinarily considered to exist outside of time; yet by adding time into their equation, Grim and his colleagues found a way to solve the age-old paradox. Their solution functions much like a light switch that contains two contradictory states (on and off), which cannot co-exist but can oscillate over time (Marks-Tarlow, 2008).

A more complicated, interpersonal variation of the Liar's paradox exists, also known since antiquity: "Socrates asserts, "Plato speaks falsely," while Plato counters, "Socrates speaks truly." To visualize the interpersonal variation of the Liar's paradox, Grim's group used fuzzy logic to supply an infinite-valued scale between truth and falsity applied to the following assertions:

x: x is as true as y;
y: y is as true as x is false

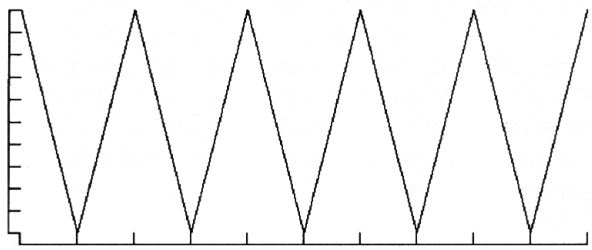

Figure 13. The simple liar's paradox. (Courtesy of Patrick Grim, Group for Logic and Formal Semantics, Department of Philosophy, SUNY at Stony Brook

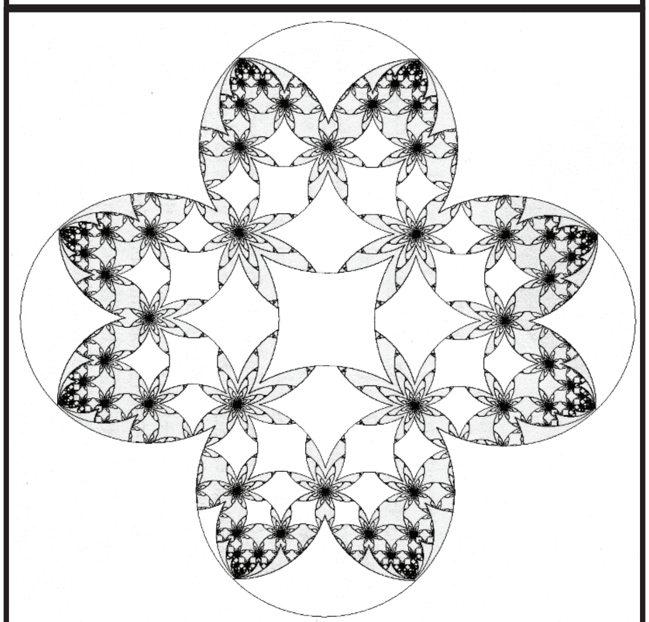

Figure 14. Interpersonal variation of the Liar's paradox. (Courtesy of Patrick Grim, Group for Logic and Formal Semantics, Department of Philosophy, SUNY at Stony Brook)

When converted into mathematical equations that were iterated by computer, figure 14 reveals the resulting fractal escape diagram. Computer modeling of interpersonal dynamics demonstrates yet again how fractal boundaries arise out of complex feedback loops between inner and outer processes, such as self and other. Grim and his colleagues, in fact, offer up fractal geometry as a means for modeling not just paradoxical logic, but in fact, all formal systems.

Epistemological Principles for Transpersonal Psychology

Having explained fractals and given examples of how they are constructed and how they have been used at the edges of psychological research, this final section offers fractal geometry as an epistemological framework for transpersonal phenomena. I would like to propose the following principles:

- Fractal geometry models and bridges recursive patterns in space, time, and the imagination;
- Fractal geometry offers quantitative methods for revealing qualitative patterns in nature previously deemed too complex, irregular, or discontinuous from the perspective of linear lenses and reductionist techniques;
- Fractal geometry models hidden as well as higher dimensional phenomena that exist in the infinite expanses *between* ordinary, finite (Euclidean) dimensions;
- Fractal dimensionality captures key features of the structure of subjective experience, such as the endless feeling of contemplation, the boundary-crossing experience of consciousness as it leaps from inner to outer worlds, and the paradox of full engagement, such that the closer we look at something, whether inside or outside the imagination, the more there is to see;
- Fractal geometry highlights idiosyncratic, non-repeatable, and rare events, by offering power law statistical distributions over time;
- Power law distributions present an alternative to normative statistics in which variability is preserved, while unpredictable, chance events

in the form of an underlying growth or decay algorithm;

- Fractal measurement illuminates observer dependence, whereby what we see depends upon how we look, including our scale of observation plus other qualities of ourselves as measuring devices;
- Fractal geometry presents a way to conceptualize fuzzy, irresolvably complex borders between various realms, levels, and dimensions of existence, including full interpenetration as it exists at fractal boundary zones;
- Fractal edges model paradoxical insights related to traditional mystical experiences and nondual states of awareness, including how the whole of things can be enfolded within the parts of existence, plus Buddhist notions of emptiness and interbeing.

I conclude this paper with a plea for transpersonal psychology to adopt a fractal epistemology. As a result, researchers will be better equipped to model idiosyncratic, rare, and unpredictable phenomena. Meanwhile, both qualitative and quantitative aspects of nature can be simulated within a single, mathematically rigorous umbrella. As a clinician and theoretician, it is my hope and vision that the adoption of a fractal epistemology might help to heal the divide between transpersonal psychologists by eliminating any need to divide the field into more versus less rigorous subfields.

References

Abbott, A. (2017). Fractal secrets of Rorschach's famed ink blots revealed. *Nature Magazine*, February 15, 2017. Retrieved from https://www.scientificamerican.com/article/fractal-secrets-of-rorschach-rsquo-s-famed-ink-blots-revealed

Baish, J., & Jain, R. (2000). Fractals and cancer. *Cancer Research*, *60*(14), 3683–3688.

Chalmers, D. *(1995).* Facing up to the problem of consciousness. *Journal of Consciousness Studies, 2*(3), 200–219.

Crick, F., & Koch, C. (1990). Toward a neurobiological theory of consciousness. *Seminars in the Neurosciences, 2*, 263–275.

Daniels, M. (1998). Paranormal psychology and the paranormal. *Transpersonal Psychology Review*, 2(3), 17–31.

Dauben, J. (1979), *Georg Cantor: His mathematics and philosophy of the infinite*. Harvard University Press.

Delignières, D., Fortes, M., & Ninot, G. (2004). Fractal dynamics of self-esteem and physical self. *Nonlinear Dynamics in Psychology and Life Sciences, 8*, 479–510.

Dewdney, A. K. (1985). A computer microscope zooms in for a look at the most complex object in mathematics. *Scientific American*, *253*(2), 16–24. https://doi.org/10.1038/scientificamerican0885-16

Ferrer, J. (2014). Transpersonal psychology, science, and the supernatural. *The Journal of Transpersonal Psychology*, *46*(2), 152–186.

Friedman, H. (1983). The Self-Expansiveness Level form: A conceptualization and measurement of a transpersonal construct. *The Journal of Transpersonal Psychology*, *15*, 37–50.

Friedman, H. (2002). Transpersonal psychology as a scientific field. *International Journal of Transpersonal Studies*, *21*, 175–181. https://doi.org/10.24972/ijts.2002.21.1.175

Friedman, H. (2013). The role of science in transpersonal psychology: The advantages of middle-range theory. In H. L. Friedman & G. Hartelius (Eds.), *The Wiley-Blackwell handbook of transpersonal psychology* (pp. 203–222). Wiley Blackwell. https://doi.org/10.1002/9781118591277.ch16

Gardner, H. (1982). *Art, mind, and brain: A cognitive approach to creativity*. Basic Books.

Grim, P., Mar, G., & St. Denis, P. (1998). The philosophical computer: Exploratory essays in philosophical computer modeling. MIT Press.

Grof, S. (2008). Brief history of transpersonal psychology. *International Journal of Transpersonal Studies*, *27*, 46–54. https://doi.org/10.24972/ijts.2008.27.1.46

Kelso, S. (1997). *Dynamic patterns*. MIT Press.

Kelso, S., & Engstrøm, D. (2006). *The complementary nature.* MIT Press.

Kelso, S., Dumas, G., & Tognoli, G. (2013). Outline of a general theory of behavior and brain coordination. *Neural Networks, 37*, 120–131. https://doi.org/10.1016/j.neunet.2012.09.003

Lajoie, D., & Shapiro, S. (1992). Definitions of transpersonal psychology: The first twenty-three years. *Journal of Transpersonal Psychology, 24*(1), 79–98.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Lawlor, R. (1982). *Sacred geometry: Philosophy & practice (Art & imagination)*. Crossroad.

Liebovitch, L. (1998) *Fractals and chaos simplified for the life sciences*. Oxford University Press.

Losa, G. (2014). On the fractal design of human brain and nervous tissue. *Applied Mathematics*, *5*, 1725–1732. https://doi.org/10.4236/am.2014.512165

Mandelbrot, B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, *156*, 636–638. https://doi.org/10.1126/science.156.3775.636

Mandelbrot, B. (1977). *The fractal geometry of nature*. W. H. Freeman.

Mandelbrot, B., & Hudson, R. (2010). *The (mis) behaviour of markets: A fractal view of risk, ruin and reward*. Profile Books.

Marks-Tarlow, T. (1999). The self as a dynamical system. *Nonlinear Dynamics, Psychology, & Life Sciences*, *3*(4), 311–345. https://doi.org/10.1023/A:1021958829905

Marks-Tarlow, T. (2004). Semiotic seams: Fractal dynamics of reentry. *Cybernetics & Human Knowing, 11*(1), 49–62.

Marks-Tarlow, T. (2008). *Psyche's veil: Psychotherapy, fractals and complexity*. Routledge.

Marks-Tarlow, T. (2010). Fractal self at play. *American Journal of Play*, *3*(1), 31–62.

Marks-Tarlow, T. (2011). Merging and emerging: A nonlinear portrait of intersubjectivity during psychotherapy. *Psychoanalytic Dialogues, 21*(1), 110–127. https://doi.org/10.1080/10481885.2011.545333

Marks-Tarlow, T. (2012). Fractal geometry as a bridge between realms. In F. Orsucci & N. Sala (Eds.), *Complexity science, living systems, and reflexing interfaces: New models and perspectives*. IGI Global.

Marks-Tarlow, T. (2015). The nonlinear dynamics of clinical intuition. *Chaos & Complexity Letters, 8*(2–3), 1–24.

Maslow, A. (1971). *The farther reaches of human nature*. Viking.

Olson, S. (2006). *The golden section: Nature's greatest secret*. Walker.

Penrose, R. (1994). *Shadows of the mind*. Oxford University Press.

Schroeder, M. (1991). *Fractals, chaos, power laws*. Freeman.

Walsh, R. (1992). The search for synthesis: Transpersonal psychology and the meeting of East and West, psychology and religion, personal and transpersonal. *Journal of Humanistic Psychology*, *32*(1), 19–45. https://doi.org/10.1177/0022167892321003

West, B. (2013). *Fractal physiology and chaos in medicine* (2nd ed.). World Scientific. https://doi.org/10.1142/8577

West, B. (2016). *Simplifying complexity: Life is uncertain, unfair and unequal*. Bentham Science.

Wigner, E. (1960). The unreasonable effectiveness of mathematics in the natural sciences. Richard courant lecture in mathematical sciences delivered at New York University, May 11, 1959. *Communications on Pure and Applied Mathematics*, *13*(1), 1–14.

Wilber, K. (1983). *Eye to eye: The quest for the new paradigm.* Anchor/Doubleday.

Williams, T., & Reilly, R. (2016). What paint can tell us: A fractal analysis of neurological changes in seven artists, *Neuropsychology, 31*(1), 1–10. https://doi.org/10.1037/neu0000303

About the Author

Terry Marks-Tarlow, PhD, is a Clinical Psychologist in private practice in Santa Monica, California. She is also an Adjunct Professor at Pacifica Graduate Institute, Santa Barbara, and Core Faculty at the Insight Center, Los Angeles. She has authored and edited several books, including: *A Fractal Epistemology for a Scientific Psychology: Bridging the Personal with the Transpersonal, Play & Creativity in Psychotherapy, Clinical Intuition in Psychotherapy, Awakening Clinical Intuition*, and *Psyche's Veil*, all of which she illustrated herself. Along with clinical practice, writing, and family life, she happily immerses herself in the arts, including dance, yoga, and writing opera librettos, one of which premiered at Lincoln Center.

About the Journal

The *International Journal of Transpersonal Studies* is a is a peer-reviewed academic journal in print since 1981. It is sponsored by the California Institute of Integral Studies, published by Floraglades Foundation, and serves as the official publication of the International Transpersonal Association. The journal is available online at www.transpersonalstudies.org, and in print through www.lulu.com (search for IJTS).